AIP Conference Proceedings

Investigation of a velocity field for the Marangoni shear convection of a vertically swirling viscous incompressible fluid

Cite as: AIP Conference Proceedings **2053**, 040011 (2018); https://doi.org/10.1063/1.5084449
Published Online: 19 December 2018

N. V. Burmasheva, and E. Yu. Prosviryakov

View Online Export Citation

ARTICLES YOU MAY BE INTERESTED IN

Investigation of temperature and pressure fields for the Marangoni shear convection of a vertically swirling viscous incompressible fluid
AIP Conference Proceedings **2053**, 040012 (2018); https://doi.org/10.1063/1.5084450

AIP | Conference Proceedings
Get 30% off all print proceedings!
Enter Promotion Code PDF30 at checkout

AIP Publishing

© 2018 Author(s).

Investigation of a Velocity Field for the Marangoni Shear Convection of a Vertically Swirling Viscous Incompressible Fluid

N. V. Burmasheva[1, 2, a)] and E. Yu. Prosviryakov[1, b)]

[1]*Institute of Engineering Science, Ural Branch of the Russian Academy of Sciences, 34 Komsomolskaya St., Ekaterinburg, 620049, Russian Federation.*
[2]*Ural Federal University named after the First President of Russia B. N. Yeltsin, 19 Mira St., Ekaterinburg, 620002, Russian Federation.*

[a)]Corresponding author: nat_burm@mail.ru
[b)]evgen_pros@mail.ru

Abstract. The article considers a new exact solution for describing large-scale flows of a vertically swirling fluid initiated by thermocapillary forces acting on a free surface. The behavior of the velocity field is analyzed. It is shown that the topology of this field depends on the values of the given parameters. It is also shown that the components of the velocity field can have several stagnant points, as a result of which the specific kinetic energy has a substantially nonmonotonic behavior.

INTRODUCTION

Investigation of fluid flows with allowance for the vertical component of vorticity is a recent problem for geophysical hydrodynamics [1-6]. In the case of solid-state fluid rotation, the value of the vertical twist is equal to the planetary vorticity of the Earth. The exact solutions of the Navier-Stokes equations describing the isobaric large-scale shear flows of a vertically swirling fluid without rotation were discussed in [7-9]. It was shown that taking into account the vertical component of the vorticity illustrates a new way of propagation of an impulse in a fluid. These exact solutions can be used to simulate the equatorial countercurrents of the World Ocean. In [10, 11] studies of the effects of horizontal pressure gradients on the structure of countercurrents for large-scale viscous incompressible fluid flows began. In [7-11] a linear distribution of the vertical twist over the thickness of the fluid layer was considered. That exact solution of the system of the Navier-Stokes equations was used for the exact integration of the Oberbeck-Boussinesq system [11]. In this paper, we consider new exact solutions for investigating the Marangoni convection of large-scale flows of vertically swirling fluids. The exact solutions presented in this paper generalize the results presented in [11-26].

BOUNDARY VALUE PROBLEM FORMULATION

The exact solutions of the system of equations of thermal convection in the Boussinesq approximation are investigated in the article. The Oberbeck-Boussinesq system used to describe shear flows of a viscous incompressible fluid in the field of gravitational forces has the following form [11,27,28]:

$$V_x \frac{\partial V_x}{\partial x} + V_y \frac{\partial V_x}{\partial y} + V_z \frac{\partial V_x}{\partial z} = \nu \Delta V_x - \frac{\partial P}{\partial x},$$

Mechanics, Resource and Diagnostics of Materials and Structures (MRDMS-2018)
AIP Conf. Proc. 2053, 040011-1–040011-6; https://doi.org/10.1063/1.5084449
Published by AIP Publishing. 978-0-7354-1781-6/$30.00

$$V_x \frac{\partial V_y}{\partial x} + V_y \frac{\partial V_y}{\partial y} + V_z \frac{\partial V_y}{\partial z} = \nu \Delta V_y - \frac{\partial P}{\partial y},$$
$$\frac{\partial P}{\partial z} = g\beta \Delta T , \quad (1)$$
$$V_x \frac{\partial T}{\partial x} + V_y \frac{\partial T}{\partial y} = \chi \Delta T ,$$
$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} = 0 .$$

Here, $V_x(x,y,z)$, $V_y(x,y,z)$ are the x - and y - components of the velocity vector $\mathbf{V}(x,y,z)$; $P(x,y,z)$ is a deviation from hydrostatic pressure, divided by the average density; $T(x,y,z)$ is the deviation of temperature from the reference value; ν, χ is the kinematic (molecular) viscosity of the fluid and the coefficient of thermal diffusivity; is the three-dimensional Laplace operator [27,28].

The solution is sought in the class [6-11]

$$V_x = U(z) + u(z)y , \; V_y = V(z), \quad (2)$$
$$T = T_0(z) + T_1(z)x + T_2(z)y , \; P = P_0(z) + P_1(z)x + P_2(z)y \quad (3)$$

By substituting the class (2)–(3) describing the motion of a vertically swirling fluid in the absence of predetermined external rotation, we arrive at the system of ordinary differential equations

$$u''(z) = 0 , \; T_1''(z) = 0 , \; P_1''(z) = g\beta T_1(z), \; \chi T_2''(z) = u(z)T_1(z), \; P_2''(z) = g\beta T_2(z), \; \nu V''(z) = P_2(z),$$
$$\nu U''(z) = V(z)u(z) + P_1(z), \; \chi T_0''(z) = U(z)T_1(z) + V(z)T_2(z), \; P_0''(z) = g\beta T_0(z). \quad (4)$$

As a system of boundary conditions, we consider the following system describing the thermocapillary effect at the upper (free) boundary $z=h$ [6-10, 13-22, 27]:

$$u(0) = \Omega , \; u'(0) = 0 , \; T_0(0) = 0 , \; T_1(0) = A , \; T_2(0) = B , \; T_0(h) = \theta , \; T_1(h) = C , \; T_2(h) = D ,$$
$$P_0(h) = S , \; P_1(h) = 0 , \; P_2(h) = 0 , \; U(0) = W \sin\alpha , \; V(0) = W \cos\alpha , \quad (5)$$
$$\eta U'(h) = -\sigma T_1(h), \; \eta V'(h) = -\sigma T_2(h).$$

We assume that the lower boundary is absolutely solid. Without loss of generality, we set $S=0$. We now consider the special case $C=D=0$ of heating only the lower boundary. Note that, with this method of specifying thermal sources at the boundaries, taking into account the thermocapillary effect is equivalent to specifying zero shearing stresses at the upper boundary of the horizontal infinite fluid layer.

EQUATION SYSTEM SOLUTION

After the normalization of the solution of system (4)–(5) to the number Ωl, the velocity field acquires the form

$$u = 1 ,$$
$$U = \frac{2\,\mathrm{Re}\sin\alpha}{Ta} - \frac{bGr\delta^3}{12Ta} Z(Z-2)(Z^2 - 2Z - 2) -$$

$$-\frac{bGr\delta^5}{720}Z(Z-2)(Z^4-4Z^3+7Z^2-6Z-12)+\frac{\operatorname{Re}\cos\alpha\delta^2}{2}Z(Z-2)-$$
$$-\frac{Gr\Pr Ta\operatorname{Re}\sin\alpha\delta^7}{241920}Z(Z-2)(3Z^6-18Z^5+20Z^4+40Z^3-130Z^2+264),$$
$$V=\frac{2\operatorname{Re}\cos\alpha}{Ta}-\frac{bGr\delta^3}{12Ta}Z(Z-2)(Z^2-2Z-2)-$$
$$-\frac{bGr\delta^5}{720}Z(Z-2)(Z^4-4Z^3+2Z^2-4Z-7).$$

Here,

$$\operatorname{Re}=\frac{Wl}{\nu},\ Gr=\frac{Cg\beta l^4}{\nu^2},\ \Pr=\frac{\nu}{\chi},\ Ta=\frac{2\Omega l^2}{\nu}$$

are the Reynolds, Grasgof, Prandtl and modified Taylor numbers respectively; δ is a geometric anisotropy parameter, $z\in(0,1)$ is a dimensionless lateral coordinate; $b=B/A$ is a dimensionless parameter.

INVESTIGATION OF THE SOLUTION

Let us analyze the resulting velocity field. We start with the component V_y representing the function $V(Z)$ in the form

$$V=\frac{2\operatorname{Re}\cos\alpha}{Ta}-\frac{bGr\delta^3}{12Ta}f_1(Z)-\frac{Gr\Pr\delta^5}{720}f_2(Z). \tag{6}$$

Thus, the velocity $V(Z)$ is determined by the superposition of three flows of different nature. The first term in expression (6) is the isothermal Couette solution [29]. The polynomials $f_1(Z)$, $f_2(Z)$ are strictly monotonic on the interval under study; however, due to their strong nonlinearity, adding a flow corresponding to either of these two polynomials to the isothermal solution may lead to the appearance of an additional stagnation point. One can strictly show that the function $V(Z)$ can have up to two stagnant points (Fig. 1).

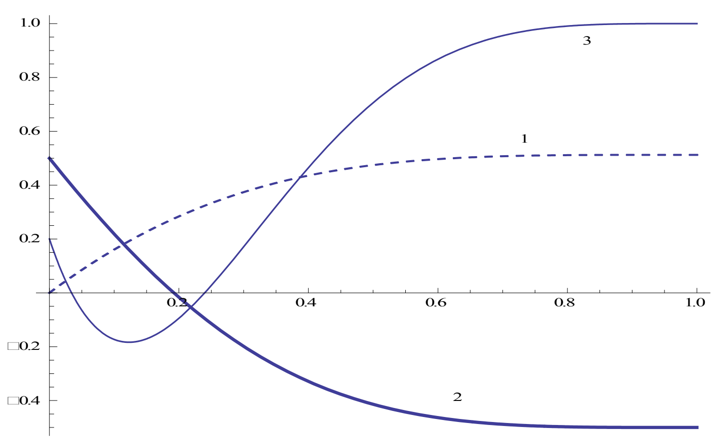

FIGURE 1. The behavior of the velocity $V(Z)$: no stagnant points (line 1), one stagnant point (line 2) and two stagnant points (line 3).

Let us now consider the behavior of the velocity $U(Z)$. We write it in the form

$$U = \frac{2\operatorname{Re}\sin\alpha}{Ta} - \frac{bGr\delta^3}{12Ta} g_1(Z) - \frac{bGr\delta^5}{720} g_2(Z) - \frac{Gr\Pr Ta\operatorname{Re}\sin\alpha\delta^7}{241920} g_3(Z) + \frac{\operatorname{Re}\cos\alpha\delta^2}{2} g_4(Z).$$

The flow *U(Z)* is a superposition of five different flows. The sum of the first and last flows is an isothermal solution. The addition of any of the remaining flows, due to the nonlinearity of the corresponding polynomial, may lead to the appearance of an additional point, at which the velocity *U(Z)* vanishes inside the layer. One can rigorously show that the number of such points for the velocity *U(Z)* does not exceed four. The behavior of the velocity *U(Z)* in each of the five corresponding situations is shown in Figs. 2 and 3.

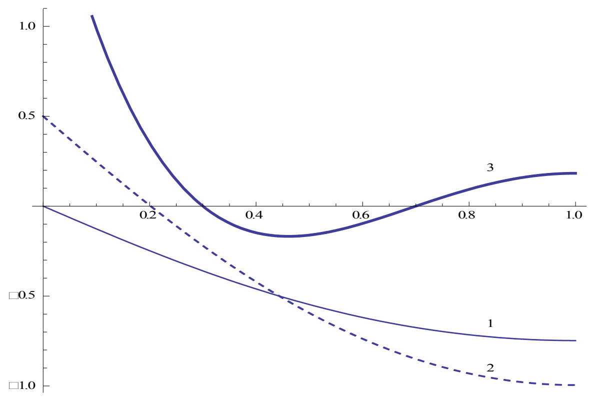

FIGURE 2. The behavior of the velocity *U(Z)*: no stagnant points (line 1), one stagnant point (line 2) and two stagnant points (line 3)

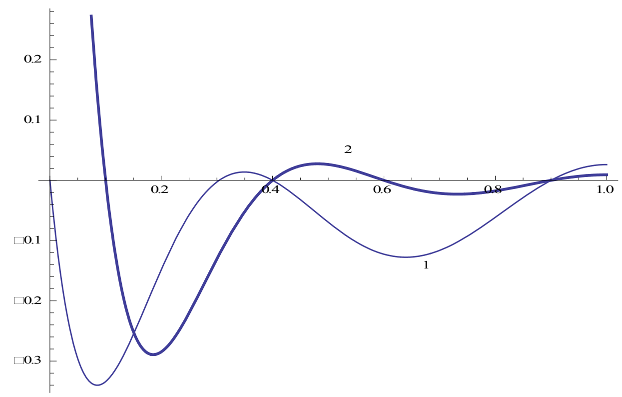

FIGURE 3. The behavior of the velocity *U(Z)*: three stagnant points (line 1), four stagnant points (line 2) and five stagnant points (line 3)

Figure 4 shows all the three hodographs of the velocity vector (for *Y=0*) for the situation when the velocity *U(Z)* admits three stagnant points. Figure 5 shows the cross sections of the surfaces of the specific kinetic energy (at *Y=0*) for the case of the presence of three stagnant points for the velocity *U(Z)*.

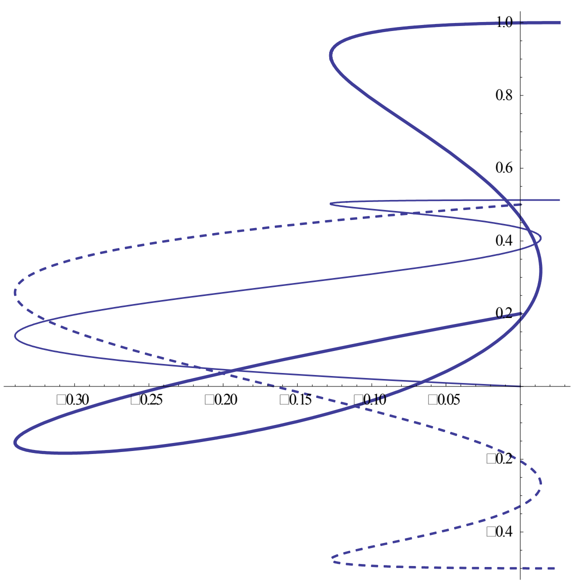

FIGURE 4. The hodographs of the velocity vector (for *Y=0*) for the case of the existence of three stagnant points for the velocity *U(Z)* (depending on the number of stagnant points for the velocity *V(Z)*).

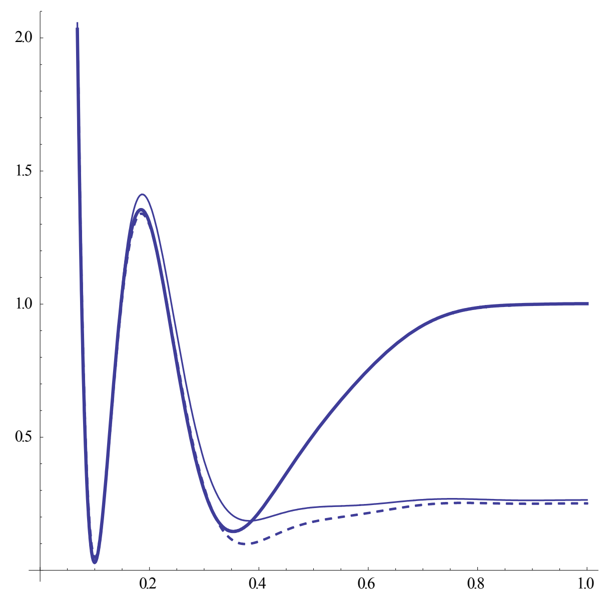

FIGURE 5. The cross sections (for *Y=0*) of the surfaces of the specific kinetic energy for the case of the existence of four stagnant points for the velocity *U(Z)* (depending on the number of stagnant points for the velocity *V(Z)*).

The figures illustrate that, in the flow of a fluid, a velocity field bundle can be observed due to the appearance of stagnant points. The stratification of the velocity field leads to the formation of countercurrents. The countercurrents occur due to the vertical twist of the fluid and its uneven heating/cooling. In this case, the specific kinetic energy has a nonmonotonic distribution, which testifies to the presence of the phenomenon of increasing velocities in the fluid. Note that, in comparison with the exact solutions published in [13-16], taking into account the vertical component of vorticity in the absence of rotation leads to a significant complexity in the topology of the velocity field.

CONCLUSION

A new exact solution for describing the convective Marangoni flow of vertically swirling fluid has been proposed. When a temperature source is specified at the lower boundary of an infinite layer, the tangential stresses on the free boundary of the layer vanish. The consideration of the vertical twist of the fluid and the zero tangential stresses lead to the formation of a very complex flow structure. It has been shown that the flow of a viscous incompressible fluid can be delaminated into four zones.

ACKNOWLEDGMENTS

This work was supported by the Foundation for Assistance to Small Innovative Enterprises in Science and Technology (the UMNIK program, agreement 12281GU/2017).

REFERENCES

1. V. W. Ekman, Arch. Math. Astron. Phys. **2** (11), 1–53 (1905).
2. J. Pedlosky, *Geophysical Fluid Dynamics, second edition* (Springer-Verlag, New York, 1987).
3. B. M. Boubnov, G. S. Golitsyn, *Convection in Rotating Fluids* (Kluwer Academic Publishers, Dordrecht, 1995).
4. D. Salmon, *Lectures on Geophysical Fluid Dynamics* (Oxford University Press, 1988).
5. V. Shtern, *Counterflows. Paradoxical Fluid Mechanics Phenomena* (Cambridge University Press, 2012).
6. S. N. Aristov, E. Yu. Prosviryakov, Nelineinaya Dinamika **10** (2), 177–182 (2014).
7. S. N. Aristov, E. Yu. Prosviryakov, Russian Aeronautics **58**(4), 413 (2015).
8. S. N. Aristov, E. Yu. Prosviryakov, Fluid Dynamics **51**(2) 148 (2016).
9. E. Yu. Prosviryakov, CEUR Workshop Proceedings **1825**, 164 (2016).
10. E. Yu. Prosviryakov, “Waves of pressure in viscous incompressible fluid,” in *Mechanics, Resource and Diagnostics of Materials and Structures, MRDMS 2017,* AIP Conference Proceedings 1915, edited by E. S. Gorkunov, V. E. Panin, S. Ramasubbu (American Institute of Physics, Melville, NY, 2017), pp. 020006.
11. S. N. Aristov, E. Yu. Prosviryakov, Fluid Dynamics **51**(5), 581 (2016).
12. S. N. Aristov, E. Yu. Prosviryakov, L. F. Spevak, Theor. Found. Chem. Eng. **50** (2), 132–141 (2016).
13. N. V. Burmasheva, E. Yu. Prosviryakov, IOP Conference Series: Materials Science and Engineering **208** (1), (2017).
14. N. V. Burmasheva, E. Yu. Prosviryakov, Vestn. Samar. Gos. Tekhn. Univ., Ser. Fiz.–Mat. Nauki **21** (1), 180–196 (2017).
15. N. V. Burmasheva, E. Yu. Prosviryakov, Vestn. Samar. Gos. Tekhn. Univ., Ser. Fiz.–Mat. Nauki **21** (4), 736–751 (2017).
16. N. V. Burmasheva, E. Yu. Prosviryakov, “Exact solution for the layered convection of a viscous incompressible fluid at specified temperature gradients and tangential forces on the free boundary,” in *Mechanics, Resource and Diagnostics of Materials and Structures, MRDMS 2017,* AIP Conference Proceedings 1915, edited by E. S. Gorkunov, V. E. Panin, S. Ramasubbu (American Institute of Physics, Melville, NY, 2017), pp. 040005. DOI: 10.1063/1.5017353.
17. E. Yu. Prosviryakov, L. F. Spevak, “Simulation of a viscous flow in layered composites in view of the thermocapillary effect,” in *Mechanics, Resource and Diagnostics of Materials and Structures, MRDMS 2017,* AIP Conference Proceedings 1915, edited by E. S. Gorkunov, V. E. Panin, S. Ramasubbu (American Institute of Physics, Melville, NY, 2017), pp. 040047. DOI: 10.1063/1.5017395.
18. L. G. Napolitano, Acta Astronautica **7** (4), 461–478 (1980).
19. L. Kh. Ingel, S.N. Aristov, Proceedings of Institute of Experimental Meteorology **27** (162), 142–157 (1996).

20. S. N. Aristov, I. M. Gitman, Journal of Fluid Mechanics **464**, 209-215 (2002).
21. O. N. Goncharova, O. A. Kabov, Microgravity Science and Technology **21** (1), 129–137 (2009).
22. K. G. Shvarz, Fluid Dynamics **49** (4), 438–442 (2014).
23. V. K. Andreev, V. B. Bekezhanova, Journal of Applied Mechanics and Technical Physics **54** (2), 171–184 (2013).
24. V. K. Andreev, Ya. A. Gaponenko, O. N. Goncharova, and V. V. Pukhnachev, *Mathematical Models of Convection* (De Gryuter Publ., Berlin–Boston, 2012), 417 p.
25. V. V. Pukhnachev, in *Application of Mathematics in Technical and Natural Sciences*, edited by M. D. Todorov and C. I. Christov (American Institute of Physics, New York, 2011), pp. 31–42.
26. K. G. Shvarts, Itogi Nauki i Tekh. Ser.: Sovrem. Mat. i Prilozh. Temat. Obz. **132**, 158–161 (2017).
27. G. Z. Gershuni, E. M. Zhukhovitskii, *Convective Stability of Incompressible Fluids* (Wiley/Keter Press, Jerusalem, 1976).
28. L. D. Landau, E M. Lifshitz, *Fluid Mechanics* (Pergamon Press, Oxford, 1987).
Couette M. ´Etudes sur le frottement des liquids. Ann. Chim. Phys. **21** (6), 433–510 (1890).